Effects of surface diffusion on the Eden model

C. S. Ryu
Electronics and Telecommunications Research Institute, Yusong P.O. Box 106, Taejon 305-600, Korea

In-mook Kim
Department of Physics, Korea University, Seoul 136-701, Korea
(Received 4 December 1995)

We study the kinetic roughening in depositions on perimeter sites. For this purpose, we study solid-on-solid models, modifications of the Eden model which was proposed for describing biological growth of cell colonies. We consider surface diffusion in vertical and horizontal directions and calculate the surface width and the correlation function, and show the surface morphologies. In the presence of only a vertical diffusion or both vertical and horizontal diffusion to local energy minima, the growth models show the Kardar-Parisi-Zhang behaviors. In contrast to these, the Edwards-Wilkinson behavior dominates the model with both vertical and horizontal diffusion to local height minima. For these results, we also give an argument based on the calculation of the step heights. [S1063-651X(96)03206-0]

PACS number(s): 05.40.+j, 81.10.Aj, 05.70.Ln, 81.15.Hi

I. INTRODUCTION

During the past decade, much attention has been paid to the kinetic roughening of growing surfaces. The surfaces grown in far-from-equilibrium conditions have been found to be self-affine, that is, invariant under an anisotropic scaling and have been investigated in sedimentation, vapor deposition, molecular beam epitaxy, bacteria colony formation, paper towels immersed into liquids, etc. [1]. An important quantity in the kinetic roughening is the surface width W, the root-mean-square value of the surface fluctuation, which has been expected to obey a finite-size scaling (FSS) proposed by Family and Vicsek [2]:

$$W(L,t)\equiv[\langle(h-\langle h\rangle)^2\rangle]^{1/2}\sim L^{\alpha}f(t/L^{z}), \tag{1}$$

where $h(\mathbf{x},t)$ is the height of the surface in $d=d'+1$ dimension (d' is the substrate dimension), L the lateral size of the substrate, t the growth time, α the roughness exponent describing a saturated surface, z the dynamic exponent, and the scaling function $f(x)\sim x^{\beta}$ (with the growth exponent $\beta=\alpha/z$) for $x\ll 1$ and $f(x)\rightarrow$const for $x\gg 1$. Here $\langle\cdots\rangle$ denotes a spatial average. Thus the surface width W grows as $W(t)\sim t^{\beta}$ for $1\ll t\ll L^{z}$ and $W(L)\sim L^{\alpha}$ for $t\gg L^{z}$.

Under the scheme of the FSS, a great deal of work to describe the surface roughness has been carried out in analytic calculations of continuum growth equations and in numerical simulations of kinetic growth models [3]. The critical exponents α and β obtained from the FSS determine the universality class of continuum equations and growth models. Representative universality classes are the Edwards-Wilkinson (EW) class [4] and the Kardar-Parisi-Zhang (KPZ) class [5] expressed as the following continuum equation:

$$\frac{\partial h}{\partial t}=\nu\nabla^2 h+\frac{\lambda}{2}(\nabla h)^2+\eta, \tag{2}$$

where η is a white noise and the equation with $\lambda=0$ corresponds to the EW equation. It has been known that the EW equation describes the surface growth under gravitation and that the random deposition with surface diffusion (RDSD) [6] belongs to this class. The KPZ equation has been considered to describe the surface growth which is locally normal to the surface (the Eden model [7] and the ballistic deposition [8]) or which occurs in the presence of a restriction on the heights of the neighboring sites [the restricted solid-on-solid (SOS) model [9]].

The Eden model, one of the earliest works, was proposed to describe the biological growth of cell colonies. In the Eden model, particles fall *on perimeter sites* (unoccupied nearest neighbors of occupied sites) with equal probability (version A in Ref. [10]), which leads to a locally normal growth to the surface. On the other hand, in most of the growth models, particles randomly fall *on substrates* and diffuse according to given growth rules. Two representative growth rules regarding surface diffusion may be those employed in the RDSD and in the Wolf-Villain (WV) model [11]. The RDSD, where freshly landed particles relax into local height minima, is described by the EW equation while the WV model, where they relax into local energy minima, shows complex crossover behaviors, despite the simple growth rule [12]; the WV model has been considered to be described by the equation $\partial h/\partial t=\nu\nabla^2 h-\nu_1\nabla^4 h+\lambda_1\nabla^2(\nabla h)^2+\eta$.

In this work, we consider depositions on perimeter sites, that is, the Eden model with surface diffusion. In other words, the incident particle flux is not uniform over a substrate but dependent on local slopes of a surface. To our knowledge, there have been few works investigating the Eden model with surface diffusion or depositions with such nonuniform incident fluxes. In our growth models, particles which randomly fall not on substrates but on perimeter sites diffuse in vertical and horizontal directions according to the growth rules employed in the RDSD and the WV model. In Sec. II we introduce our growth models. In Sec. III we present numerical results and discussions. Section IV is devoted to a brief summary.

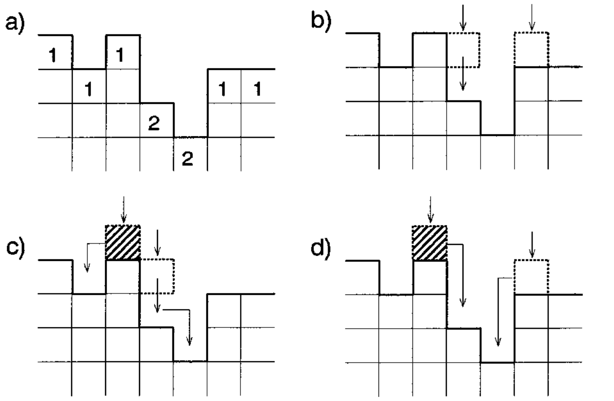

FIG. 1. Deposition rate at each site of the substrate (a) and schematic growth rules of models A (b), B (c), and C (d). One can notice the difference between models B and C in the move of the dashed particle.

II. THE GROWTH MODELS

In our growth models, particles randomly fall on perimeter sites. Thus, in view of the substrate, the deposition rate is not the same at each site of the substrate. A simple instance is illustrated in Fig. 1(a). First, we define model A as the Eden model only with vertical diffusion. Next, we consider additional diffusion after the vertical diffusion; particles diffuse within nearest-neighbor columns according to two kinds of growth rules (models B and C). We note that our growth models are all SOS models. The growth rules of models A, B, and C are given in detail as follows.

Model A. A freshly landed particle on a perimeter site diffuses only in a vertical direction and moves to the top of the deposit, as shown in Fig. 1(b).

Model B. After the vertical diffusion, the particle is allowed to diffuse again. It moves to local energy minimum within nearest-neighbor interaction, as in the WV model. See Fig. 1(c).

Model C. After the vertical diffusion, the particle moves to local height minimum as in the RDSD, instead of local energy minimum. See Fig. 1(d).

In the next section, numerical simulations on one-dimensional substrates show that models A and B can be described by the KPZ equation ($\alpha = 1/2$ and $\beta = 1/3$) and model C by the EW equation ($\alpha = 1/2$ and $\beta = 1/4$).

III. RESULTS AND DISCUSSIONS

In this section, we present numerical results of growth models A, B, and C on one-dimensional substrates. In simulations, we use periodic boundary conditions. First, we calculate the surface width W for three growth models. Figure 2 shows the log-log plot of W vs t for model A. We have $\beta = 0.326 \pm 0.001$ very close to 1/3. We also have $\alpha = 0.507 \pm 0.001$ very close to 1/2 from the log-log plot of W vs L as shown in the inset. The values of α and β suggest that model A is described by the KPZ equation. Since model A differs from the Eden model only in allowing the vertical diffusion to the top of the deposit, there is no possibility leading to other derivatives of $h(x)$ in the KPZ equation. Thus we arrive at the conclusion that model A belongs to the KPZ class. As shown in the figure, the convergence to the asymptotic behavior is relatively fast, compared to the original Eden model. We consider that the fast convergence is due to the suppression of vacancies and overhangs achieved by the vertical diffusion, as by application of noise reduction [13]. We note that SOS models such as model A and the restricted SOS model show faster convergence than the Eden model and the ballistic deposition allowing vacancies and overhangs.

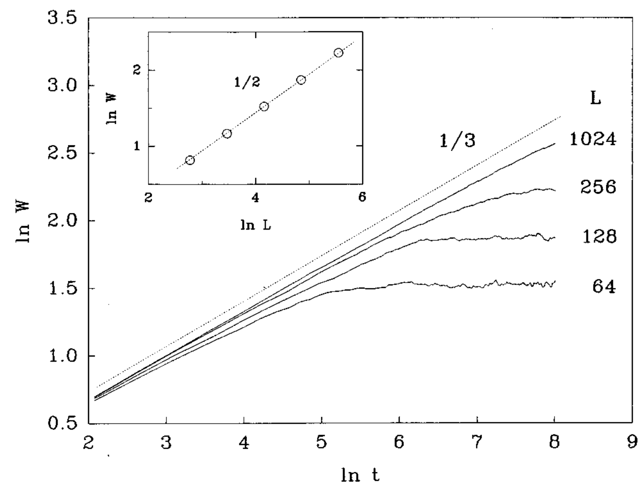

FIG. 2. The log-log plots of W vs t for model A. The slope of the guide dotted-line is 1/3. The inset shows the log-log plot of W vs L where the slope of the guide dotted-line is 1/2. Statistical averages were taken on 500 samples.

For model B, we obtain $\alpha = 0.492 \pm 0.002$ and $\beta = 0.323 \pm 0.001$, as shown in Fig. 3. The values of α and β are close to the analytic values of the KPZ equation. As shown in the figure, model B shows slower convergence than model A. To confirm the results obtained from the surface width, we also calculate the height-difference correlation function $G(r,t)$ obeying the following scaling ansatz:

$$G(r,t) \equiv \langle [h(x+r,t) - h(x,t)]^2 \rangle \sim r^{2\alpha} g(r/t^{1/z}), \tag{3}$$

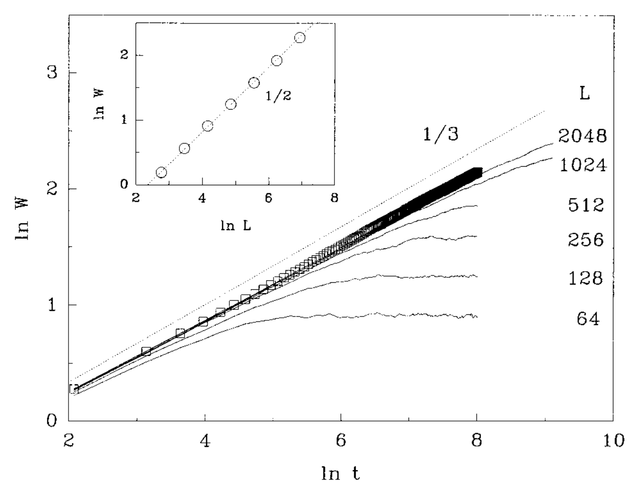

FIG. 3. The log-log plots of W vs t for model B, where the slope of the guide dotted-line is 1/3. We obtained β for $L = 4096$ (□). The inset shows the log-log plot of W vs L where the slope of the guide dotted-line is 1/2. Statistical averages were taken on 400 to 1000 samples.

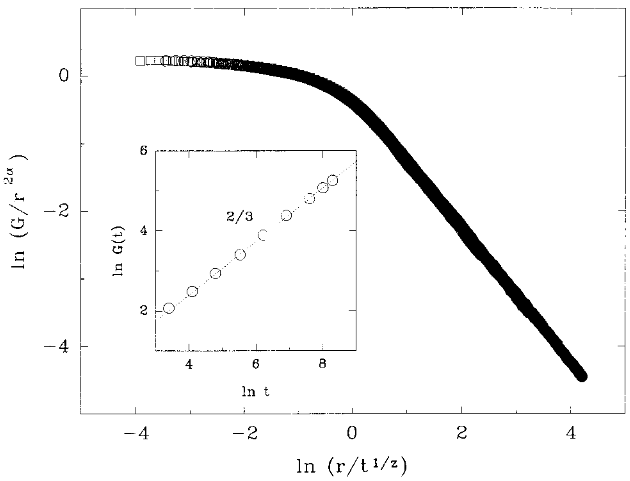

FIG. 4. The scaling plots of $G(r,t)$ for $t=60$, 120, 250, 500, 1000, 2000, and 4000 with $L=2048$. We have $\alpha=1/2$ and $z=3/2$. Statistical averages were taken on 100 samples. The inset shows the log-log plot of G vs t, where the slope 2β of the guide dotted-line is 2/3.

where the scaling function $g(x)\rightarrow$const for $x \ll 1$ and $g(x)\sim x^{-2\alpha}$ for $x \gg 1$. Thus the correlation function G grows as $G(r)\sim r^{2\alpha}$ for $r \ll t^{1/z}$ and $G(t)\sim t^{2\beta}$ for $r \gg t^{1/z}$. As shown in the inset of Fig. 4, we obtain $\beta=0.331\pm0.002$ from the log-log plot of $G(t)$ vs t, where $G(t)$ was obtained from the saturated values of $G(r,t)$ in the plots of $G(r,t)$ vs r. We also show the scaling plots of $G(r,t)$ with $\alpha=1/2$ and $z=3/2$ in the figure. The perfect data collapse confirms that model B belongs to the KPZ class. It has been known that the growth rule of the WV model leads to other higher-order terms such as $-\nu_1\nabla^4 h$ and $\lambda_1\nabla^2(\nabla h)^2$. Since the nonlinear $(\nabla h)^2$ term is the most relevant term in the renormalization group sense, one can expect that model B shows the KPZ behavior.

Next we investigate model C. In contrast to models A and B, model C shows the EW behavior. As shown in Fig. 5, we obtained $\alpha=0.484\pm0.002$ and $\beta=0.248\pm0.001$ very close to the analytic values of the EW equation. This somewhat surprising result can be understood in the following way. Uniform incident flux on perimeter sites can be different from uniform flux on the substrate, if and only if there exist steps with heights greater than unity. In model C, those steps do not develop owing to the diffusion to local height minima. To validate this argument, we calculate $G(1,t)=\langle(\nabla h)^2\rangle$ in saturated regimes.

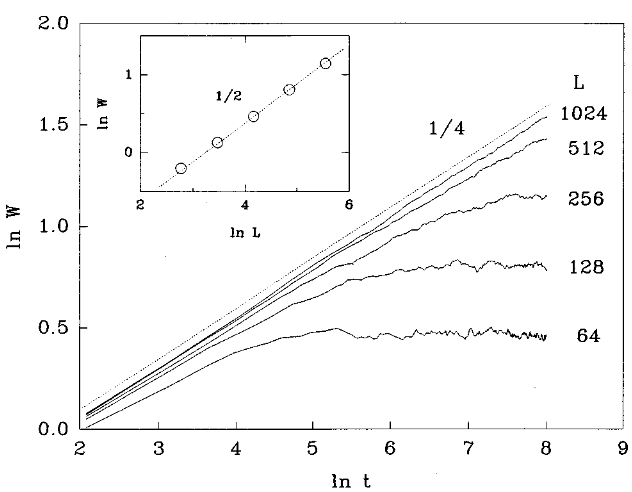

FIG. 5. The log-log plots of W vs t for model C. The slope of the guide dotted-line is 1/4. The inset shows the log-log plot of W vs L where the slope of the guide dotted-line is 1/2. Statistical averages were taken on 400 to 500 samples.

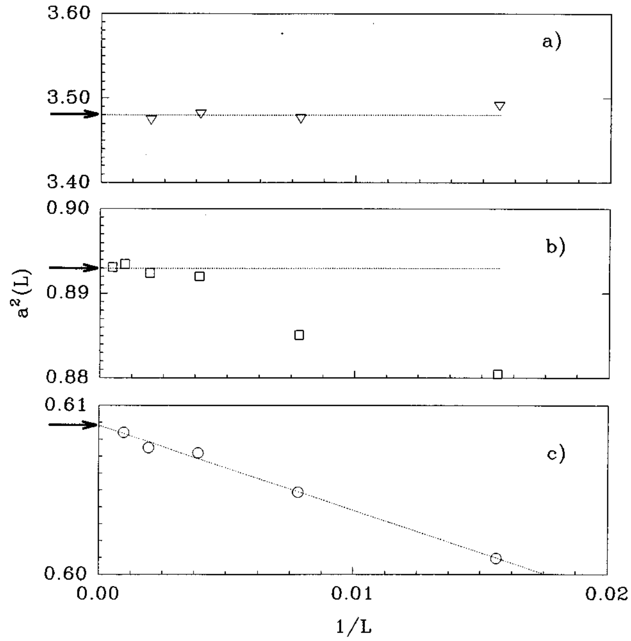

FIG. 6. The figure shows the plots of $G(1,t)=a^2$ vs $1/L$ in saturated regimes for models A (a), B (b), and C (c). The arrows indicate the asymptotic values of a^2, where root-mean-square values of step heights are $a\sim2$ for model A, ~1 for model B, and ~0.8 for model C. Statistical averages were taken on 300 to 500 samples.

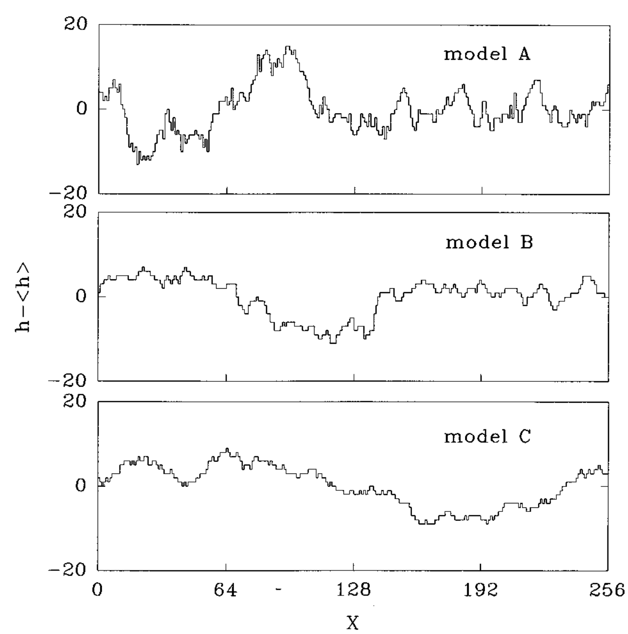

FIG. 7. The surface morphologies at $t=10^5$ for models A, B, and C.

As shown in Fig. 6(a), the root-mean-square step height a, where $G(1,t\rightarrow\infty)\equiv a^2$, is ≈ 1.9 and independent of L for model A. This confirms the KPZ behavior of the model. For model C, as shown in Fig. 6(c), we have $a^2(L)\sim c_1-c_2/L$ ($a\approx 0.78$), where $c_1\approx 0.609$ and $c_2\approx 0.5$. In the EW class, the saturated values of $G(1,t)$ are constant with a correction of order $1/L$ [14]. This validates our argument for the EW behavior of model C. In model B, $a\approx 0.95$ and $a^2(L)$ does not show clear dependence on L. Considering flat terraces on the surface, $a\sim 1$ implies the presence of large step-heights. Thus we confirm the KPZ behavior of model B.

We also show the surface morphologies. In model A, the surface morphology clearly shows large step-heights which can be found on any part of the surface. For $L=256$, 42% of the sites have step-heights larger than unity. In model B, a deep valley is observed as in the WV model and 10% of the sites have step-heights larger than unity, which is considered to be enough to yield the KPZ behavior. In contrast to models A and B, one can observe a smooth surface invariant under the transformation $h\rightarrow -h$ for model C. Moreover, only seven sites have $|\nabla h|>1$. It is considered that this value less than 3% of total sites is not generic but due to finite-size effect. Thus for model C, there is no difference between the deposition on perimeter sites and that on the substrate, which leads to the EW behavior of model C. By calculating $G(1,t)$ and showing the surface morphologies, we have confirmed the KPZ behaviors of models A and B and the EW behavior of model C.

IV. SUMMARY

To investigate the kinetic roughening in depositions on perimeter sites, we have studied three growth models as the Eden model with surface diffusion. In the presence of vertical diffusion (model A) and additional diffusion to local energy minima (model B), the models are considered to belong to the Kardar-Parisi-Zhang (KPZ) class as the original Eden model without any diffusion process. Allowing diffusion to local height minima instead of local energy minima (model C), the surface roughness can be described by the Edwards-Wilkinson (EW) equation. To show these behaviors, we have calculated the surface width, the correlation function, and the averaged step-height and shown the surface morphologies. The KPZ behaviors of models A and B are due to the growth locally normal to surfaces. For model C, where few sites have step heights larger than unity, the deposition on perimeter sites does not differ from that on the substrate, and thus the model shows the EW behavior.

ACKNOWLEDGMENTS

C.S.R. is supported by the Ministry of Information and Communication, Korea and is very grateful to Dr. E. H. Lee for his support of this work. I.M.K. is supported in part by KOSEF (Project No. 951-0206-003-2) and the Ministry of Education (Project No. BSRI-95-2409), Korea.

[1] T. Vicsek, *Fractal Growth Phenomena*, 2nd ed. (World Scientific, Singapore, 1992), and references therein.
[2] F. Family and T. Vicsek, J. Phys. A **18**, L75 (1985).
[3] *Dynamics of Fractal Surfaces*, edited by F. Family and T. Vicsek (World Scientific, Singapore, 1991); J. Krug and H. Spohn, in *Solids Far From Equilibrium: Growth, Morphology and Defects*, edited by C. Godreche (Cambridge University Press, New York, 1991).
[4] S. F. Edwards and D. R. Wilkinson, Proc. R. Soc. London, Ser. A **381**, 17 (1982).
[5] M. Kardar, G. Parisi, and Y. C. Zhang, Phys. Rev. Lett. **56**, 889 (1986).
[6] F. Family, J. Phys. A **19**, L441 (1986).
[7] M. Eden, in *Proceedings of the Fourth Berkeley Symposium on Mathematics Statistics and Probability*, edited by F. Neyman (Univ. of Calif. Press, Berkeley, 1961), Vol. 4, p. 223.
[8] P. Meakin, P. Ramanlal, L. M. Sander, and R. C. Ball, Phys. Rev. A **34**, 5091 (1986).
[9] J. M. Kim and J. M. Kosterlitz, Phys. Rev. Lett. **62**, 2289 (1989).
[10] R. Jullien and R. Botet, J. Phys. A **18**, 2279 (1985).
[11] D. E. Wolf and J. Villain, Europhys. Lett. **13**, 389 (1990).
[12] S. Das Sarma and S. V. Ghaisas, Phys. Rev. Lett. **69**, 3762 (1992); J. Krug, M. Plischke, and M. Siegert, *ibid.* **70**, 3271 (1993); M. Kotrla, A. Č. Levi, and P. Šmilauer, Europhys. Lett. **20**, 25 (1992); P. Šmilauer and M. Kotrla, Phys. Rev. B **49**, 5769 (1994); C. S. Ryu and I. M. Kim, Phys. Rev. E **51**, 3069 (1995); **52**, 2424 (1995).
[13] J. Kertész and D. E. Wolf, J. Phys. A **21**, 747 (1988).
[14] K. Park, B. Kahng, and S. S. Kim, Physica A **210**, 146 (1994).